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A full-invariant theorem and some applications

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Abstract

Let (X,τ_1) and (Y,τ_2) be two Hausdorff locally convex spaces with continuous duals X' and Y', respectively, $L(X,Y)$ be the space of all continuous linear operators from X into Y, $K(X,Y)$ be the space of all compact operators of $L(X,Y)$. Let *WOT* and *UOT* be the weak operator topology and uniform operator topology on $K(X,Y)$, respectively. In this paper, we characterize a full-invariant property of $K(X,Y)$; that is, if the sequence space λ has the signed-weak gliding hump property, then each λ-multiplier *WOT*-convergent series $\sum_i T_i$ in $K(X,Y)$ must be λ-multiplier convergent with respect to all topologies between *WOT* and *UOT* if and only if each continuous linear operator $T:(X,\tau_1)\to(\lambda^\beta,\sigma(\lambda^\beta,\lambda))$ is compact. It follows from this result that the converse of Kalton's Orlicz–Pettis theorem is also true. © 2002 Elsevier Science (USA). All rights reserved.

Keywords: Locally convex space; Sequence space; Compact operator; Full-invariant

1. Introduction

Let (X,τ_1) and (Y,τ_2) be two Hausdorff locally convex spaces. An operator $T\in L(X,Y)$ is said to be compact if for each bounded subset A of (X,τ_1), $T(A)$ is a relatively compact subset of (Y,τ_2). Let $K(X,Y)$ denote the set of all compact operators of $L(X,Y)$.

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Let *WOT*, *SOT* and *UOT* be the weak operator topology, strong operator topology and uniform operator topology on $K(X, Y)$; i.e., $\lim_\alpha T_\alpha = 0$ in the *WOT* $\Leftrightarrow$ for each $x \in X$, $y' \in Y'$, $\lim_\alpha \langle T_\alpha x, y'\rangle = 0$; $\lim_\alpha T_\alpha = 0$ in the *SOT* $\Leftrightarrow$ for each $x \in X$, $\lim_\alpha T_\alpha(x) = 0$; $\lim_\alpha T_\alpha = 0$ in the *UOT* $\Leftrightarrow$ for each bounded subset A of X, $\lim_\alpha T_\alpha x = 0$ uniformly with respect to $x \in A$.

Let ω be the space of all scalar valued sequences. A non-zero sequence $\{z^{(n)}\}$ in ω is said to be a block sequence if there exists a strictly increasing sequence $\{k_n\}$ of integers with $k_0 = 0$ such that

$$z^{(n)} = \left(0, 0, \ldots, 0, z^{(n)}_{k_{n-1}+1}, \ldots, z^{(n)}_{k_n}, 0, \ldots\right).$$

A space λ of scalar-valued sequences is said to have the signed-weak gliding hump property (s-wghp) if, given any $t = (t_i) \in \lambda$ and any block sequence $\{t^{(k)}\}$ with $t = \sum_{k=1}^\infty t^{(k)}$ (pointwise sum), each index sequence $\{m_k\}$ has a further subsequence $\{n_k\}$ and a signed sequence $\{s_k\}$ with $s_k = 1$ or $s_k = -1$ $(k \in N)$, such that $\tilde{t} = \sum_{k=1}^\infty s_k t^{(n_k)} \in \lambda$ (pointwise sum) [1].

A space λ of scalar-valued sequences is said to be a monotone space provided that the coordinate product $x, y \in \lambda$ whenever $x \in m_0$ and $y \in \lambda$, where m_0 is the space of all scalar sequence $x = (x_i)$ such that $\{x_i\colon i \in N\}$ is a finite set [2].

Any monotone space has the s-wghp, while the sequence space

$$bs = \left\{ (t_i)\colon \sup_n \left| \sum_{i=1}^n t_i \right| < \infty \right\}$$

has the s-wghp, but fails to be a monotone space [1].

A series $\sum_i x_i$ in (X, τ_1) is said to be subseries τ_1-convergent if for each strictly increasing positive integers sequence $\{n_i\}$, the series $\sum_i x_{n_i}$ is τ_1-convergent.

A series $\sum_i x_i$ in (X, τ_1) is said to be λ-multiplier τ_1-convergent if for each $t = (t_i) \in \lambda$, the series $\sum_i t_i x_i$ is τ_1-convergent.

It is obvious that subseries convergent is equivalent to m_0-multiplier convergent.

If the series $\sum_i x_i$ is l^∞-multiplier τ_1-convergent, then $\sum_i x_i$ is said to be bounded-multiplier convergent.

As is known, studying the invariant property is a crucial problem in locally convex space theory. The Mackey theorem shows that boundedness is a duality-invariant property; the Mazur theorem shows that the closed convexity is a duality-invariant property [3]. The Orlicz–Pettis theorem shows that the subseries convergent is also a duality-invariant property [4].

Recently, Li Ronglu and Cui Chengri showed that if $\lambda = c_0$ or $\lambda = l^p$ $(1 \leqslant p < \infty)$, then the series $\sum_i x_i$ in (X, τ_1) is λ-multiplier convergent with respect to the weak topology $\sigma(X, X')$ if and only if $\sum_i x_i$ is λ-multiplier convergent with respect to all admissible (X, X')-polar topologies; i.e., the c_0-multiplier convergent and the l^p-multiplier convergent $(1 \leqslant p < \infty)$ of series are invariant with respect to all admissible topologies. Note that this kind of full-invariance is rare in the theory of locally convex spaces, except c_0-multiplier convergence, l^p-multiplier convergence ($1 \leqslant p < \infty$) of series and some trivial facts (cf. [5]).

In this paper, we characterize a full-invariant property of $K(X, Y)$. From it we deduce that the converse of Kalton's Orlicz–Pettis theorem is also valid.

Let c_{00} be the scalar-valued sequence space which are 0 eventually, the β-dual space of λ to be defined by $\lambda^\beta = \{u = (u_i) \in \omega\colon \sum_i u_i t_i$ is convergent for each $(t_i) \in \lambda\}$. It is obvious that if $c_{00} \subseteq \lambda$, then λ and λ^β are in duality with respect to the bilinear map $\langle t, u\rangle = \sum_i u_i t_i$, $t = (t_i) \in \lambda$, $u = (u_i) \in \lambda^\beta$. The weak topology of λ from this pairing is denoted by $\sigma(\lambda, \lambda^\beta)$, a similar notation is used for the weak topology of λ^β.

2. Characterization of a full-invariant property

A sequence $\{t^{(n)}\}_{n=1}^\infty$ in ω is said to be coordinatewise convergent to $t^{(0)}$ if for each $i \in N$, $\{t_i^{(n)}\}_{n=1}^\infty$ converges to $t_i^{(0)}$.

The following Lemma 1 is important in this paper (for its proof see [6, p. 415(1)] or [7, p. 31]).

Lemma 1. *Let $c_{00} \subseteq \lambda$ and $M \subseteq \lambda^\beta$. The following statements are equivalent*:

(1) *M is $\sigma(\lambda^\beta, \lambda)$-compact*;
(2) *M is $\sigma(\lambda^\beta, \lambda)$-sequentially compact*;
(3) *M is $\sigma(\lambda^\beta, \lambda)$-bounded, and each sequence $\{t^{(n)}\} \subseteq M$, which is coordinatewise convergent to $t^{(0)} \in \omega$, must be $\sigma(\lambda^\beta, \lambda)$ convergent to $t^{(0)}$ and $t^{(0)} \in M$.*

From Lemma 1 and [1, Theorem 5(iii)] or [8, proof of Theorem 3.6], we can prove the following:

Lemma 2. *Let $c_{00} \subseteq \lambda$, and λ have the s-wghp, $A \subseteq \lambda^\beta$ be a $\sigma(\lambda^\beta, \lambda)$-relatively compact set. Then for each $t = (t_i) \in \lambda$, the series $\sum_i t_i u_i$ converges uniformly with respect to $u = (u_i) \in A$.*

Theorem 1. *Let (X, τ_1) and (Y, τ_2) be two Hausdorff locally convex spaces and $Y \neq \{0\}$, $c_{00} \subseteq \lambda$, and λ have the s-wghp. Then each λ-multiplier WOT-convergent series $\sum_i T_i$ in $K(X, Y)$ is λ-multiplier convergent with respect to all topologies between WOT and UOT if and only if each continuous linear operator*

$$T : (X, \tau_1) \to \bigl(\lambda^\beta, \sigma\bigl(\lambda^\beta, \lambda\bigr)\bigr)$$

is compact.

Proof. Suppose that the series $\sum_i T_i$ in $K(X,Y)$ is λ-multiplier *WOT*-convergent; that is, for each $t=(t_i)\in\lambda$, there exists a $T_0\in K(X,Y)$ such that for $x\in X$ and $y'\in Y'$, we have

$$\sum_i t_i\langle T_i x, y'\rangle = \langle T_0 x, y'\rangle.$$

First, we show that for each $x\in X$, the series $\sum_i t_i T_i x$ is τ_2-convergent to $T_0 x$; i.e., the series $\sum_i T_i$ is λ-multiplier *SOT*-convergent. Note that the topology τ_2 is weaker than the Mackey topology $\tau(Y,Y')$, so for each $x\in X$, if we can prove that the series $\sum_i t_i T_i x$ is $\tau(Y,Y')$-convergent to $T_0 x$, then $\sum_i t_i T_i x$ must be τ_2-convergent to $T_0 x$. If for some $x\in X$, the series $\sum_i t_i T_i x$ is not $\tau(Y,Y')$-convergent to $T_0 x$, then there exists an absolutely convex $\sigma(Y',Y)$-compact subset B of Y' and $\varepsilon_0>0$ such that for each $i_0\in N$, there exists an $i_1\in N$, $i_1>i_0$ and $y'\in B$ such that $|\langle\sum_{i=1}^{i_1} t_i T_i x - T_0 x, y'\rangle|\geqslant\varepsilon_0$, or equivalently,

$$\left|\left\langle \sum_{i=i_1+1}^{\infty} t_i T_i x, y'\right\rangle\right| \geqslant \varepsilon_0. \tag{1}$$

This shows that the series $\sum_{i=1}^{\infty} t_i\langle T_i x, y'\rangle$ does not converge uniformly with respect to $y'\in B$. Note that for each $(t_i)\in\lambda$, the series $\sum_{i=1}^{\infty} t_i\langle T_i x, y'\rangle$ is convergent, so $(\langle T_i x, y'\rangle)_{i=1}^{\infty}\in\lambda^{\beta}$. On the other hand, since $\sum_{i=1}^{\infty} t_i\langle T_i x, y'\rangle = \langle T_0 x, y'\rangle$, the mapping $y'\to(\langle T_i x, y'\rangle)_{i=1}^{\infty}$ is a continuous linear operator $(Y',\sigma(Y',Y))\to(\lambda^{\beta},\sigma(\lambda^{\beta},\lambda))$. Since B is a $(Y',\sigma(Y',Y))$-compact subset of Y', $\{(\langle T_i x, y'\rangle)_{i=1}^{\infty}\colon\ y'\in B\}$ is a $\sigma(\lambda^{\beta},\lambda)$-compact subset of λ^{β}. It follows from Lemma 2 that the series $\sum_{i=1}^{\infty} t_i u_i$ converges uniformly with respect to $(u_i)\in\{(\langle T_i x, y'\rangle)_{i=1}^{\infty}\colon\ y'\in B\}$. That is, $\sum_{i=1}^{\infty} t_i\langle T_i x, y'\rangle$ converges uniformly with respect to $y'\in B$. This contradicts (1) and, therefore, the series $\sum_i T_i$ is λ-multiplier *SOT*-convergent.

Sufficiency. Let the series $\sum_i T_i$ in $K(X,Y)$ be λ-multiplier *WOT*-convergent, so $\sum_i T_i$ must be also λ-multiplier *SOT*-convergent. Without loss of generality, we only need to show that $\sum_i T_i$ is λ-multiplier *UOT*-convergent. If not, there is a bounded subset A of (X,τ_1), $t^{(0)}=(t_i^{(0)})\in\lambda$ and $T_0\in K(X,Y)$ such that for each $x\in A$, the series $\sum_i t_i^{(0)} T_i x$ is τ_2-convergent to $T_0 x$, but $\sum_i t_i^{(0)} T_i x$ does not convergent to $T_0 x$ uniformly with respect to $x\in A$. Thus, there are a continuous seminorm p of (Y,τ_2) and $\varepsilon_0>0$ such that for each $k\in N$, there are $m,n\in N$, $m\geqslant n>k$ and $x\in A$ with

$$p\left(\sum_{i=n}^{m} t_i^{(0)} T_i x\right)\geqslant\varepsilon_0.$$

For $k=1$, there exist $m_1\geqslant n_1>1$ and $x_1\in A$ with $p(\sum_{i=n_1}^{m_1} t_i^{(0)} T_i x_1)\geqslant\varepsilon_0$. For $k=m_1$, there exist $m_2\geqslant n_2>m_1$ and $x_2\in A$ with $p(\sum_{i=n_2}^{m_2} t_i^{(0)} T_i x_2)\geqslant\varepsilon_0$.

Inductively, we can obtain sequences $n_1 \leqslant m_1 < n_2 \leqslant m_2 < \cdots < n_k \leqslant m_k < \cdots$ in N and $x_k \in A$ such that

$$p\left(\sum_{i=n_k}^{m_k} t_i^{(0)} T_i x_k\right) \geqslant \varepsilon_0, \quad k \in N.$$

By the Hahn–Banach theorem, there is a sequence $\{y'_k\}$ of Y' such that for each $k \in N$, $\|y'_k\|_p = \sup_{p(x)\leqslant 1} |y'_k(x)| \leqslant 1$ and

$$y'_k\left(\sum_{i=n_k}^{m_k} t_i^{(0)} T_i x_k\right) \geqslant \varepsilon_0, \quad k \in N.$$

Let Y_0 be the linear closed hull of $\{T_k x_n\colon k, n \in N\}$ in (X, τ_2). Then (Y_0, p) is a separable seminormed space. Thus, we can obtain a subsequence $\{y'_{k_j}\}$ of $\{y'_k\}$, without loss of generality, we may assume that $\{y'_{k_j}\}$ is just $\{y'_k\}$, and $y'_0 \in Y'$ with $\|y'_0\|_p \leqslant 1$ such that for each $y \in Y_0$, $\lim_k y'_k(y) = y'_0(y)$.

For each $T \in K(X, Y)$, let T' be the adjoint operator of T; that is, $T' : Y' \to X'$ and for $x \in X$, $y' \in Y'$, T' satisfies that

$$\langle Tx, y'\rangle = \langle x, T'y'\rangle.$$

For $x' \in X'$, denote $q(x') = \sup_n\{|x'(x_n)|\}$. Then q is a $\beta(X', X)$-continuous seminorm since $\{x_n\colon n \in N\}$ is bounded. Now, we show that if $\{Tx_n\} \subseteq Y_0$, then

$$\lim_k q\left(T'y'_k - T'y'_0\right) = 0.$$

If not, there exist a subsequence $\{y'_{k_j}\}$ of $\{y'_k\}$, a sequence $\{x_{k_j}\} \subseteq \{x_n\}$ and $\varepsilon_1 > 0$ such that

$$\left|\left(T'y'_{k_j} - T'y'_0\right)(x_{k_j})\right| \geqslant \varepsilon_1, \quad j \in N;$$

i.e.,

$$\left|\left(y'_{k_j} - y'_0\right)(Tx_{k_j})\right| \geqslant \varepsilon_1, \quad j \in N. \tag{2}$$

On account of $T \in K(X, Y)$, the set $\{Tx_{k_j}\}$ is relatively compact in (Y, τ_2). Note that p is a continuous seminorm on (Y, τ_2) and $\{Tx_{k_j}\} \subseteq Y_0$, so $\{Tx_{k_j}\}$ is a relatively compact subset of the seminormed space (Y_0, p), and is also a relatively sequentially compact subset of (Y_0, p). Thus, without loss of generality, we may assume that there exists a $y_0 \in Y_0$ such that $\{p(Tx_{k_j} - y_0)\}$ converges to 0. Note that

$$\begin{aligned}
\left|\left(y'_{k_j} - y'_0\right)(Tx_{k_j})\right| &\leqslant \left|\left(y'_{k_j} - y'_0\right)(Tx_{k_j} - y_0)\right| + \left|\left(y'_{k_j} - y'_0\right)(y_0)\right| \\
&\leqslant \left\|y'_{k_j} - y'_0\right\|_p p(Tx_{k_j} - y_0) + \left|\left(y'_{k_j} - y'_0\right)(y_0)\right|.
\end{aligned}$$

It follows from $\|y'_{k_j} - y'_0\|_p \leqslant 2$, and from the fact that $\{p(Tx_{k_j} - y_0)\}$ converges to 0 and $\{y'_{k_j}(y_0)\}$ converges to $y'_0(y_0)$, that

$$\lim_j \bigl(y'_{k_j} - y'_0\bigr)(Tx_{k_j}) = 0.$$

This contradicts (2). Therefore, for each $T \in K(X,Y)$, if $\{Tx_n\} \subseteq Y_0$, then

$$\lim_k q\bigl(T'y'_k - T'y'_0\bigr) = 0.$$

Furthermore, if $t = (t_i) \in \lambda$ and the series $\sum_i t_i T_i$ is *SOT*-convergent to T, then for $y' \in Y'$ and $x \in X$, $\sum_i t_i \langle T_i x, y'\rangle = \langle Tx, y'\rangle$. So $x \to (\langle T_i x, y'\rangle)_{i=1}^{\infty}$ is a continuous linear operator $(X, \tau_1) \to (\lambda^\beta, \sigma(\lambda^\beta, \lambda))$. It follows from the condition in Theorem 1 that for each bounded subset A of (X, τ_1), $\{(\langle T_i x, y'\rangle)_{i=1}^{\infty}\colon x \in A\}$ is a relatively $\sigma(\lambda^\beta, \lambda)$-compact subset of λ^β. Thus, it follows from Lemma 2 that for each bounded subset A of (X, τ_1) and $y' \in Y'$, the series $\sum_{i=1}^{\infty} t_i \langle T_i x, y'\rangle$ converges to $\langle Tx, y'\rangle$ uniformly with respect to $x \in A$. Or equivalently, for each $y' \in Y'$, the series $\sum_{i=1}^{\infty} t_i T'_i y'$ in X' is $\beta(X', X)$-convergent to $T'y'$.

Now, we consider the infinite matrix $[\sum_{i=n_j}^{m_j} t_i^{(0)} T'_i y'_k]_{kj}$. For each $j \in N$, because of $\sum_{i=n_j}^{m_j} t_i^{(0)} T_i \in K(X,Y)$ and $\{\sum_{i=n_j}^{m_j} t_i^{(0)} T_i x_n\} \subseteq Y_0$, we have

$$\lim_k q\left(\sum_{i=n_j}^{m_j} t_i^{(0)} T'_i y'_k - \sum_{i=n_j}^{m_j} t_i^{(0)} T'_i y'_0\right) = 0.$$

For each strictly increasing sequence of positive integers $\{j_l\}$, by the s-wghp of λ, we can obtain a subsequence $\{j_{l_p}\}$ of $\{j_l\}$ and a signed sequence $\{s_p\} \subseteq \{-1, 1\}$ such that $\sum_{p=1}^{\infty} \sum_{i=n_{j_{l_p}}}^{m_{j_{l_p}}} s_p t_i^{(0)} \in \lambda$ (pointwise sum). Thus, there exists a $T_0 \in K(X,Y)$ such that the series $\sum_{p=1}^{\infty} \sum_{i=n_{j_{l_p}}}^{m_{j_{l_p}}} s_p t_i^{(0)} T_i$ is *SOT*-convergent to T_0; therefore, the series $\sum_{p=1}^{\infty} \sum_{i=n_{j_{l_p}}}^{m_{j_{l_p}}} s_p t_i^{(0)} T'_i y'_k$ is $\beta(X', X)$-convergent to $T'_0 y'_k$. Thus we have

$$q\left(\sum_{p=1}^{\infty} \sum_{i=n_{j_{l_p}}}^{m_{j_{l_p}}} s_p t_i^{(0)} T'_i y'_k - T'_0 y'_k\right) = 0.$$

Note that $\{T_0 x_n\} \subseteq Y_0$ is obvious; therefore, $\lim_k q(T'_0 y'_k - T'_0 y'_0) = 0$. It follows from [9, Theorem 2.24] that

$$\lim_k q\left(\sum_{i=n_k}^{m_k} t_i^{(0)} T'_i y'_k\right) = 0.$$

This contradicts (2) and the sufficiency holds.

Necessity. Let T be a continuous linear operator of $(X,\tau_1)\to(\lambda^\beta,\sigma(\lambda^\beta,\lambda))$; for $x\in X$, write $Tx=(T(x)_i)_{i=1}^\infty$. Pick $y\in Y$, $y\neq 0$, and define $T_i:X\to Y$ for $T_ix=T(x)_iy$. It is obvious that $T_i\in K(X,Y)$. Furthermore, for each $t=(t_i)\in\lambda$, let $T_0x=\langle t,Tx\rangle y$; then $T_0\in K(X,Y)$ and $\sum_i t_iT_i$ is *SOT*-convergent to T_0. That is, $\sum_i T_i$ is a λ-multiplier *SOT*-convergent series in $K(X,Y)$, and, hence, is also a λ-multiplier *WOT*-convergent series in $K(X,Y)$. It follows from the condition in Theorem 1 that $\sum_i T_i$ is a λ-multiplier *UOT*-convergent series. This shows that for each $t=(t_i)\in\lambda$ and each bounded subset A of (X,τ_1), the series $\sum_i t_iT_ix$ converges uniformly with respect to $x\in A$. For each sequence $\{Tx_n\}\subseteq\{Tx\colon x\in A\}$ we can find by the diagonal method a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$ such that $\{Tx_{n_k}\}$ is a coordinatewise convergent sequence. Since the series $\sum_i t_iT_ix_{n_k}$ converges uniformly with respect to $k\in N$, it is easy to show that $\{Tx_{n_k}\}$ is a $\sigma(\lambda^\beta,\lambda)$-Cauchy sequence. By the sequential completeness of $(\lambda^\beta,\sigma(\lambda^\beta,\lambda))$ [10, Theorem 3.10], there exists a $u=(u_i)\in\lambda^\beta$ such that $\{Tx_{n_k}\}$ is $\sigma(\lambda^\beta,\lambda)$-convergent to u. It follows from Lemma 1 that $\{Tx\colon x\in A\}$ is a relatively compact subset of $(\lambda^\beta,\sigma(\lambda^\beta,\lambda))$; i.e., T is a compact operator $(X,\tau_1)\to(\lambda^\beta,\sigma(\lambda^\beta,\lambda))$. The necessity holds and the theorem is proved. □

3. The converse of Kalton's theorem

Kalton in [11, Theorem 5] proved the following famous Orlicz–Pettis theorem:

Let $(E,\|.\|)$ and $(F,\|.\|)$ be two Banach spaces and $(E',\|.\|)$ containing no copy of l^∞. If the series $\sum_i T_i$ in $K(X,Y)$ is subseries WOT-convergent, then $\sum_i T_i$ must be subseries norm-convergent.

Since m_0 is a monotone space, it has the s-wghp. It is easy to see that $(m_0)^\beta=l^1$. It follows from the Schur lemma [3, Theorem 1.3.2 and Remark 15.2.3] that $(l^1,\sigma(l^1,m_0))$, $(l^1,\sigma(l^1,l^\infty))$ and $(l^1,\|.\|)$ have the same relatively sequentially compact sets, by Lemma 1 that $(l^1,\sigma(l^1,m_0))$, $(l^1,\sigma(l^1,l^\infty))$ and $(l^1,\|.\|)$ have the same relatively compact sets. Thus, we have

Theorem 2. *Let (X,τ_2) and (Y,τ_2) be two Hausdorff locally convex spaces and $Y\neq\{0\}$. Then each WOT-subseries convergent series $\sum_i T_i$ in $K(X,Y)$ is a UOT-subseries convergent series if and only if each continuous linear operator $T:(X,\tau_1)\to(l^1,\|.\|)$ is compact.*

Theorem 3. *Let (X,τ_2) and (Y,τ_2) be two Hausdorff locally convex spaces and $Y\neq\{0\}$. Then each bounded multiplier WOT-convergent series $\sum_i T_i$ in $K(X,Y)$ must be bounded multiplier UOT-convergent if and only if each continuous linear operator $T:(X,\tau_1)\to(l^1,\|.\|)$ is compact.*

Wu Junde and Li Ronglu in [12] showed that if (X, τ_1) is a barrelled locally convex space, then $(X', \beta(X', X))$ contains no copy of $(l^\infty, \|.\|)$ if and only if each continuous linear operator $T : (X, \tau_1) \to (l^1, \|.\|)$ is compact. By the conclusion and Theorem 1 we have:

Theorem 4. *Let (X, τ_2) and (Y, τ_2) be two Hausdorff locally convex spaces and (X, τ_1) a barrelled space, $Y \neq \{0\}$. Then each WOT-subseries convergent series $\sum_i T_i$ in $K(X, Y)$ must be UOT-subseries convergent if and only if $(X', \beta(X', X))$ contains no copy of $(l^\infty, \|.\|)$.*

Theorem 4 shows that the converse of Kalton's Orlicz–Pettis theorem is also valid.

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